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The Number of Vector Partitions of *n* (Counted According to the weight) with the Crank *m*

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ABSTRACT

This article shows how to find all vector partitions of any positive integral values of *n*, but only all vector partitions of 4, 5 and 6 are shown by algebraically. These must be satisfied by the definitions of crank of vector partitions.

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INTRODUCTION

Here we discuss such a crank which in terms of a weighted count of what we call vector partitions. We give the definitions of π, $\#(\pi)$, $\sigma(\pi)$, crank of vector partitions, weight of $\vec{\pi}$, $N_V(m,n)$, $N_V(m,t,n)$ and prove the partitions congruences moduli 5, 7 and 11 with the help of examples by finding all vector partitions of 4, 5 and 6, respectively. We analyze the generating functions for $N_V(m,n)$ and $N_V(m,t,n)$.

DEFINITIONS

π : A partition.

$\#(\pi)$: The number of parts of π.

$\sigma(\pi)$: The sum of the parts of π.

Crank of vector partitions: The number of parts of π_2 minus the number of parts of π_3,

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where π_2 and π_3 are unrestricted partitions in a vector partition $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$ of *n*, if the sum of $\vec{\pi}$ is $s(\vec{\pi}) = \sigma(\pi_1) + \sigma(\pi_2) + \sigma(\pi_3) = n$.

Weight of $\vec{\pi}$: Weight of vector partition $\vec{\pi}$ is defined as; $\omega(\vec{\pi}) = (-1)^{\#(\pi_1)}$.

$N_V(m,n)$: The number of vector partitions of *n* (counted according to the weight ω) with the crank *m*.

$N_V(m,t,n)$: The number of vector partitions of *n* (counted according to the weight ω) with the crank congruent to *m* modulo *t*.

The Crank for Vector Partitions

For a partition π, let $\#(\pi)$ be the number of parts of π and $\sigma(\pi)$ be the sum of the parts of π with the convention $\#(\phi) = \sigma(\phi) = 0$ for the empty partition ϕ of 0 (Andrews, 1985), (Andrews and Garvan, 1988).

Let, $\vec{V} = \{(\pi_1, \pi_2, \pi_3) | \pi_1$ is a partition into unequal parts π_2, π_3 are unrestricted partitions}.

We shall call the elements of $\vec{V}$ vector partitions. For $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$ in $\vec{V}$ we define the sum of parts, *s*, a weight, ω, and a crank, *c*, by;

$s(\vec{\pi}) = \sigma(\pi_1) + \sigma(\pi_2) + \sigma(\pi_3)$.

$\omega(\vec{\pi}) = (-1)^{\#(\pi_1)}$.

$c(\vec{\pi}) = \#(\pi_1) - \#(\pi_2)$.

We say $\vec{\pi}$ is a vector partition of *n*, if $s(\vec{\pi}) = n$. For example, if $\vec{\pi} = (1,1+1,1)$, then $s(\vec{\pi}) = 4$, $\omega(\vec{\pi}) = -1$, $c(\vec{\pi}) = 1$ and $\vec{\pi}$ is a vector partition of 4.

The number of vector partitions of *n* (counted according to the weight ω) with the crank *m* is denoted by $N_v(m,n)$ so that;

$N_V(m,n) = \sum \omega(\vec{\pi})$; if $\vec{\pi} \in \vec{V}$, $s(\vec{\pi}) = n$, and $c(\vec{\pi}) = m$.

We have 41 vector partitions of 4 are given in the following table:

Vector partitions of 4	Weight $\omega(\vec{\pi})$	Crank $(\vec{\pi})$
$\vec{\pi}_1 = (\phi, \phi, 4)$	+1	-1
$\vec{\pi}_2 = (\phi, \phi, 3+1)$	+1	-2
$\vec{\pi}_3 = (\phi, \phi, 2+2)$	+1	-2
$\vec{\pi}_4 = (\phi, \phi, 2+1+1)$	+1	-3
$\vec{\pi}_5 = (\phi, \phi, 1+1+1+1)$	+1	-4
$\vec{\pi}_6 = (\phi, 1, 3)$	+1	0

$\vec{\pi}_7 = (\phi, 1, 2+1)$	+1	–1
$\vec{\pi}_8 = (\phi, 1+1+1+1)$	+1	–2
$\vec{\pi}_9 = (\phi, 2+2)$	+1	0
$\vec{\pi}_{10} = (\phi, 2, 1+1)$	+1	–1
$\vec{\pi}_{11} = (\phi, 1+1, 2)$	+1	1
$\vec{\pi}_{12} = (\phi, 1+1, 1+1)$	+1	0
$\vec{\pi}_{13} = (\phi, 3, 1)$	+1	0
$\vec{\pi}_{14} = (\phi, 2+1, 1)$	+1	1
$\vec{\pi}_{15} = (\phi, 1+1+1, 1)$	+1	2
$\vec{\pi}_{16} = (\phi, 4, \phi)$	+1	1
$\vec{\pi}_{17} = (\phi, 3+1, \phi)$	+1	2
$\vec{\pi}_{18} = (\phi, 2+2, \phi)$	+1	2
$\vec{\pi}_{19} = (\phi, 2+1+1, \phi)$	+1	3
$\vec{\pi}_{20} = (\phi, 1+1+1+1, \phi)$	+1	4
$\vec{\pi}_{21} = (1, \phi, 3)$	–1	–1
$\vec{\pi}_{22} = (1, \phi, 2+1)$	–1	–2
$\vec{\pi}_{23} = (1, \phi, 1+1+1)$	–1	–3
$\vec{\pi}_{24} = (1, 1, 2)$	–1	0
$\vec{\pi}_{25} = (1, 1, 1+1)$	–1	–1
$\vec{\pi}_{26} = (1, 2, 1)$	–1	0
$\vec{\pi}_{27} = (1+1, 1, 1)$	–1	1
$\vec{\pi}_{28} = (1, 3, \phi)$	–1	1
$\vec{\pi}_{29} = (1, 2+1, \phi)$	–1	2
$\vec{\pi}_{30} = (1, 1+1+1, \phi)$	–1	3
$\vec{\pi}_{31} = (2, \phi, 2)$	–1	–1
$\vec{\pi}_{32} = (2, \phi, 1+1)$	–1	–2
$\vec{\pi}_{33} = (2, 1, 1)$	–1	0
$\vec{\pi}_{34} = (2, 2, \phi)$	–1	1
$\vec{\pi}_{35} = (2, 1+1, \phi)$	–1	2

$\vec{\pi}_{36} = (3, \phi, 1)$	–1	–1
$\vec{\pi}_{37} = (2+1, \phi, 1)$	+1	–1
$\vec{\pi}_{38} = (3, 1, \phi)$	–1	1
$\vec{\pi}_{39} = (2+1, 1, \phi)$	+1	1
$\vec{\pi}_{40} = (4, \phi, \phi)$	–1	0
$\vec{\pi}_{41} = (3+1, \phi, \phi)$	+1	0

From the above table we have,

$$N_V(0,4) = \omega(\vec{\pi}_6) + \omega(\vec{\pi}_9) + \omega(\vec{\pi}_{12}) + \omega(\vec{\pi}_{13}) + \omega(\vec{\pi}_{24}) + \omega(\vec{\pi}_{26}) + \omega(\vec{\pi}_{33}) + \omega(\vec{\pi}_{40}) + \omega(\vec{\pi}_{41})$$

$$= 1+1+1+1-1-1-1-1+1 = 1 \qquad (1)$$

The number of vector partitions of n (counted according to the weight ω) with the crank congruent to k modulo t is denoted by $N_V(k,t,n)$, so that;

$$N_V(k,t,n) = \sum_{m=-\infty}^{\infty} N_V(m, t+k, n) = \sum \omega(\vec{\pi}); \qquad (2)$$

if $\vec{\pi} \in \vec{V}$, $s(\vec{\pi}) = n$, and $c(\vec{\pi}) \equiv k(\mathrm{mod}\, t)$.

From the table we get;

$$N_V(1,5,4) = \omega(\vec{\pi}_5) + \omega(\vec{\pi}_{11}) + \omega(\vec{\pi}_{14}) + \omega(\vec{\pi}_{16}) + \omega(\vec{\pi}_{27}) + \omega(\vec{\pi}_{28}) + \omega(\vec{\pi}_{34}) + \omega(\vec{\pi}_{38}) + \omega(\vec{\pi}_{39})$$

$$= 1+1+1+1-1-1-1-1+1 = 1. \qquad (3)$$

By considering the transformation that interchanges π_2 and π_3 we have;

$$N_V(m,n) = N_V(-m,n).$$

We illustrate with an example;

$$N_V(1,4) = \omega(\vec{\pi}_{11}) + \omega(\vec{\pi}_{14}) + \ldots + \omega(\vec{\pi}_{39})$$

$= 1 + 1 + 1-1-1-1-1+1 = 0.$

and

$$N_V(-1,4) = \omega(\vec{\pi}_1) + \omega(\vec{\pi}_7) + \ldots + \omega(\vec{\pi}_{37})$$

$= 1 + 1 + 1-1-1-1-1+1 = 0$

$$\therefore\ N_V(1,4) = N_V(-1,4).$$

Again,

$$N_V(5-1,5,4) = N_V(4,5,4) = \omega(\vec{\pi}_{20}) = 1$$

$$\therefore N_V(1,5,4) = N_V(5-1,5,4) \quad \text{by (3).}$$

Generally we can write,

$$N_V(m,t,n) = N_V(t-m,t,n)$$

The Generating Function for $N_V(m,n)$

The generating function for $N_V(m,n)$ is;

$$\prod_{n=1}^{\infty}\frac{(1-x^n)}{(1-zx^n)(1-z^{-1}x^n)}=\sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty}N_V(m,n)z^n x^n \quad (4)$$

which was proved by Atkin and Swinner ton-Dyer (1954). By putting $z = 1$ in (4), we get;

$$\prod_{n=1}^{\infty}\frac{(1-x^n)}{(1-x^n)(1-x^n)}$$

$$=\sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty}N_V(m,n)x^n$$

$$\Rightarrow\sum_{n=0}^{\infty}P(n)x^n=\sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty}N_V(m,n)x^n$$

$$\therefore\ P(n)=\sum_{m=-\infty}^{\infty}N_V(m,n). \quad (5)$$

Now we discuss it with an example;

R. H. S. = $\sum_{m=-\infty}^{\infty}N_V(m,n)$

$$=\sum_{m=-\infty}^{\infty}N_V(m,4)$$

$$=\ldots+N_V(-4,4)+N_V(-3,4)+N_V(-2,4)+N_V(-1,4)+N_V(0,4)+N_V(1,4)+N_V(2,4)$$
$$+N_V(3,4)+N_V(4,4)+\ldots$$

= 0 + 1 + 0 + 1 + 0 + 1 + 0 + 1 + 0 + 1= 5 = $P(4)$= L. H. S.

The Generating Function for $N_V(0,n)$

The generating function for $N_V(0,n)$ is defined as;

$$(1-x)\sum_{n=0}^{\infty}\frac{x^{n(n+2)}}{(x^2)_n}$$

$$=(1-x)\left[1+\frac{x^3}{(1-x)^2}+\frac{x^8}{(1-x)^2(1-x^2)^2}+\frac{x^{15}}{(1-x)^2(1-x^3)^2}+\ldots\right]$$

$$=1-x+0.x^2+x^3+x^4+x^5+x^6+\ldots$$

$$=N_V(0,0)+N_V(0,1)x+N_V(0,2)x^2+N_V(0,3)x^3+N_V(0,4)x^4+N_V(0,5)x^5+N_V(0,6)x^6$$
$$+\ldots$$

$= \sum_{n=0}^{\infty} N_V(0,n)\, x^n$.

Result

- The result is;

$$N_V(k,5,5n+4) = \frac{P(5n+4)}{5};\ 0 \le k \le 4.$$

Proof: We prove the result with an example.
From the table 1 we get;

$N_V(0,5,4) = \omega(\vec{\pi}_6) + \omega(\vec{\pi}_9) + \omega(\vec{\pi}_{12}) + \omega(\vec{\pi}_{13}) + \omega(\vec{\pi}_{24}) +$
$\omega(\vec{\pi}_{26}) + \omega(\vec{\pi}_{33}) + \omega(\vec{\pi}_{40}) + \omega(\vec{\pi}_{41})$
= 1+1+1+1–1–1–1–1+1 = 1,

$N_V(1,5,4)$ = 1+1+1+1–1–1–1–1+1 = 1,

$N_V(2,5,4)$ = 1+1+1+1–1–1–1= 1,

$N_V(3,5,4)$ = 1+1+1–1–1+1–1= 1,

$N_V(4,5,4)$ = 1+1+1–1–1–1–1+1+1 = 1.

$\therefore\ N_V(0,5,4) = N_V(1,5,4) = N_V(2,5,4) = N_V(3,5,4) = N_V(4,5,4) = 1 = \frac{P(4)}{5}$, where n = 0.

In general we can write;

$$N_V(k,5,5n+4) = \frac{P(5n+4)}{5};\ 0 \le k \le 4.$$

Hence the Theorem.

- The result is;

$$N_V(k,7,7n+5) = \frac{P(7n+4)}{7};\ 0 \le k \le 6.$$

Proof: We prove the result with an example.

The vector partitions of 5 are given in the table below:

Vector partitions of 5	Weight $\omega(\vec{\pi})$	Crank $(\vec{\pi})$
$\vec{\pi}_1 = (\phi,\phi,5)$	+1	–1
$\vec{\pi}_2 = (\phi,\phi,4+1)$	+1	–2
$\vec{\pi}_3 = (\phi,\phi,3+2)$	+1	–2
$\vec{\pi}_4 = (\phi,\phi,3+1+1)$	+1	–3
$\vec{\pi}_5 = (\phi,\phi,2+2+1)$	+1	–3

$\vec{\pi}_6 = (\phi, \phi, 2+1+1+1)$	+1	–4
$\vec{\pi}_7 = (\phi, \phi, 1+1+1+1+1)$	+1	–5
$\vec{\pi}_8 = (5, \phi, \phi)$	–1	0
$\vec{\pi}_9 = (\phi, 5, \phi)$	+1	1
$\vec{\pi}_{10} = (\phi, 4+1, \phi)$	+1	2
$\vec{\pi}_{11} = (4+1, \phi, \phi)$	+1	0
$\vec{\pi}_{12} = (4, 1, \phi)$	–1	1
$\vec{\pi}_{13} = (1, 4, \phi)$	–1	1
$\vec{\pi}_{14} = (\phi, 4, 1)$	+1	0
$\vec{\pi}_{15} = (\phi, 1, 4)$	+1	0
$\vec{\pi}_{16} = (1, \phi, 4)$	–1	–1
$\vec{\pi}_{17} = (4, \phi, 1)$	–1	–1
$\vec{\pi}_{18} = (3+2, \phi, \phi)$	+1	0
$\vec{\pi}_{19} = (\phi, 3+2, \phi)$	+1	2
$\vec{\pi}_{20} = (3, 2, \phi)$	–1	1
$\vec{\pi}_{21} = (2, 3, \phi)$	–1	1
$\vec{\pi}_{22} = (\phi, 3, 2)$	+1	0
$\vec{\pi}_{23} = (\phi, 2, 3)$	+1	0
$\vec{\pi}_{24} = (3, \phi, 2)$	–1	–1
$\vec{\pi}_{25} = (2, \phi, 3)$	–1	–1
$\vec{\pi}_{26} = (\phi, 3+1+1, \phi)$	+1	3
$\vec{\pi}_{27} = (3+1, 1, \phi)$	+1	1
$\vec{\pi}_{28} = (1, 3+1, \phi)$	–1	2
$\vec{\pi}_{29} = (\phi, 3+1, 1)$	+1	1
$\vec{\pi}_{30} = (\phi, 1, 3+1)$	+1	–1
$\vec{\pi}_{31} = (3+1, \phi, 1)$	+1	–1
$\vec{\pi}_{32} = (1, \phi, 3+1)$	–1	–2
$\vec{\pi}_{33} = (3, 1+1, \phi)$	–1	2
$\vec{\pi}_{34} = (\phi, 1+1, 3)$	+1	1

$\vec{\pi}_{35} = (\phi,3,1+1)$	+1	–1
$\vec{\pi}_{36} = (3,\phi,1+1)$	–1	–2
$\vec{\pi}_{37} = (\phi,2+2+1,\phi)$	+1	3
$\vec{\pi}_{38} = (1,2+2,\phi)$	–1	2
$\vec{\pi}_{39} = (\phi,2+2,1)$	+1	1
$\vec{\pi}_{40} = (\phi,1,2+2)$	+1	–1
$\vec{\pi}_{41} = (1,\phi,2+2)$	–1	–2
$\vec{\pi}_{42} = (2+1,2,\phi)$	+1	1
$\vec{\pi}_{43} = (2,2+1,\phi)$	–1	2
$\vec{\pi}_{44} = (\phi,2,2+1)$	+1	1
$\vec{\pi}_{45} = (\phi,2+1,2)$	+1	1
$\vec{\pi}_{46} = (2+1,\phi,2)$	+1	–1
$\vec{\pi}_{47} = (2,\phi,2+1)$	–1	–2
$\vec{\pi}_{48} = (\phi,2+2+1,\phi)$	+1	4
$\vec{\pi}_{49} = (\phi,2+1+1,1)$	+1	2
$\vec{\pi}_{50} = (\phi,1,2+1+1)$	+1	–2
$\vec{\pi}_{51} = (1,2+1+1,\phi)$	–1	3
$\vec{\pi}_{52} = (1,\phi,2+1+1)$	–1	–3
$\vec{\pi}_{53} = (2+1,1+1,\phi)$	+1	2
$\vec{\pi}_{54} = (\phi,2+1,1+1)$	+1	0
$\vec{\pi}_{55} = (\phi,1+1,2+1)$	+1	0
$\vec{\pi}_{56} = (2+1,\phi,1+1)$	+1	–2
$\vec{\pi}_{57} = (\phi,1+1+1,2)$	+1	2
$\vec{\pi}_{58} = (\phi,2,1+1+1)$	+1	–2
$\vec{\pi}_{59} = (2,1+1+1,\phi)$	–1	3
$\vec{\pi}_{60} = (2,\phi,1+1+1)$	–1	–3
$\vec{\pi}_{61} = (\phi,1+1+1+1+1,\phi)$	+1	5
$\vec{\pi}_{62} = (\phi,1+1+1+1,1)$	+1	3
$\vec{\pi}_{63} = (\phi,1,1+1+1+1)$	+1	–3

$\vec{\pi}_{64} = (1, \phi, 1+1+1+1)$	–1	–4
$\vec{\pi}_{65} = (1, 1+1+1+1, \phi)$	–1	4
$\vec{\pi}_{66} = (\phi, 1+1, 1+1+1)$	+1	–1
$\vec{\pi}_{67} = (\phi, 1+1+1, 1+1)$	+1	1
$\vec{\pi}_{68} = (1, 1, 1+1+1)$	–1	–2
$\vec{\pi}_{69} = (1, 1+1+1, 1)$	–1	2
$\vec{\pi}_{70} = (1, 1+1, 1+1)$	–1	0
$\vec{\pi}_{71} = (1, 1+1, 2)$	–1	1
$\vec{\pi}_{72} = (1, 2, 1+1)$	–1	–1
$\vec{\pi}_{73} = (2, 1+1, 1)$	–1	1
$\vec{\pi}_{74} = (2, 1, 1+1)$	–1	–1
$\vec{\pi}_{75} = (2, 2, 1)$	–1	0
$\vec{\pi}_{76} = (2, 1, 2)$	–1	0
$\vec{\pi}_{77} = (1, 2, 2)$	–1	0
$\vec{\pi}_{78} = (3, 1, 1)$	–1	0
$\vec{\pi}_{79} = (1, 3, 1)$	–1	0
$\vec{\pi}_{80} = (1, 1, 3)$	–1	0
$\vec{\pi}_{81} = (1+2, 1, 1)$	+1	0
$\vec{\pi}_{82} = (1, 1+2, 1)$	–1	1
$\vec{\pi}_{83} = (1, 1, 1+2)$	–1	–1

From this table we have;

$$N_V(0,7,5) = \omega(\vec{\pi}_8) + \omega(\vec{\pi}_{11}) + \omega(\vec{\pi}_{14}) + \omega(\vec{\pi}_{15}) +$$
$$\omega(\vec{\pi}_{18}) + \omega(\vec{\pi}_{22}) + \omega(\vec{\pi}_{23}) + \omega(\vec{\pi}_{54}) + \omega(\vec{\pi}_{55}) +$$
$$\omega(\vec{\pi}_{70}) + \omega(\vec{\pi}_{75}) + \omega(\vec{\pi}_{76}) + \omega(\vec{\pi}_{78}) + \omega(\vec{\pi}_{79}) + \omega(\vec{\pi}_{79}) + \omega(\vec{\pi}_{80}) + \omega(\vec{\pi}_{81})$$

= –1+1+1+1+1+1+1+1+1–1–1–1–1–1–1–1–1–1+1 = 1.

Similarly,

$$N_V(0,7,5) = N_V(1,7,5) = \ldots = N_V(6,7,5) = 1 = \frac{P(5)}{7}.$$

In general we can write;

$$N_V(k,7,7n+5) = \frac{P(7n+5)}{7};\ 0 \le k \le 6.$$

Hence the result.

- The result is;

$$N_V\left(k,11,11n+6\right)=\frac{P(11n+6)}{11}.$$

Proof: We prove the result with an example.
The vector partitions of 6 are given in the table below:

Vector partitions of 6	Weight $\omega(\vec{\pi})$	Crank $(\vec{\pi})$
$\vec{\pi}_1=(\phi,\phi,6)$	+1	–1
$\vec{\pi}_2=(\phi,\phi,5+1)$	+1	–2
$\vec{\pi}_3=(\phi,\phi,4+2)$	+1	–2
$\vec{\pi}_4=(\phi,\phi,4+1+1)$	+1	–3
$\vec{\pi}_5=(\phi,\phi,3+3)$	+1	–2
$\vec{\pi}_6=(\phi,\phi,3+2+1)$	+1	–3
$\vec{\pi}_7=(\phi,\phi,3+1+1+1)$	+1	–4
$\vec{\pi}_8=(\phi,\phi,2+2+2)$	+1	–3
$\vec{\pi}_9=(\phi,\phi,2+2+1+1)$	+1	–4
$\vec{\pi}_{10}=(\phi,\phi,2+1+1+1+1)$	+1	–5
$\vec{\pi}_{11}=(\phi,\phi,1+1+1+1+1+1)$	+1	–6
$\vec{\pi}_{12}=(\phi,6,\phi)$	+1	1
$\vec{\pi}_{13}=(\phi,5+1,\phi)$	+1	2
$\vec{\pi}_{14}=(\phi,4+2,\phi)$	+1	2
$\vec{\pi}_{15}=(\phi,4+1+1,\phi)$	+1	3
$\vec{\pi}_{16}=(\phi,3+3,\phi)$	+1	2
$\vec{\pi}_{17}=(\phi,3+2+1,\phi)$	+1	3
$\vec{\pi}_{18}=(\phi,3+1+1+1,\phi)$	+1	4
$\vec{\pi}_{19}=(\phi,2+2+2,\phi)$	+1	3
$\vec{\pi}_{20}=(\phi,2+2+1+1,\phi)$	+1	4
$\vec{\pi}_{21}=(\phi,2+1+1+1+1,\phi)$	+1	5
$\vec{\pi}_{22}=(\phi,1+1+1+1+1+1,\phi)$	+1	6
$\vec{\pi}_{23}=(6,\phi,\phi)$	–1	0

$\vec{\pi}_{24} = (5+1, \phi, \phi)$	+1	0
$\vec{\pi}_{25} = (4+2, \phi, \phi)$	+1	0
$\vec{\pi}_{26} = (3+2+1, \phi, \phi)$	–1	0
$\vec{\pi}_{27} = (\phi, 5, 1)$	+1	0
$\vec{\pi}_{28} = (\phi, 1, 5)$	+1	0
$\vec{\pi}_{29} = (\phi, 4, 2)$	+1	0
$\vec{\pi}_{30} = (\phi, 2, 4)$	+1	0
$\vec{\pi}_{31} = (\phi, 4, 1)$	+1	1
$\vec{\pi}_{32} = (\phi, 4, 1+1)$	+1	–1
$\vec{\pi}_{33} = (\phi, 1, 4+1)$	+1	–1
$\vec{\pi}_{34} = (\phi, 1+1, 4)$	+1	1
$\vec{\pi}_{35} = (\phi, 3, 3)$	+1	0
$\vec{\pi}_{36} = (\phi, 3+2, 1)$	+1	1
$\vec{\pi}_{37} = (\phi, 1, 3+2)$	+1	–1
$\vec{\pi}_{38} = (\phi, 3, 2+1)$	+1	–1
$\vec{\pi}_{39} = (\phi, 2+1, 3)$	+1	1
$\vec{\pi}_{40} = (\phi, 1+3, 2)$	+1	1
$\vec{\pi}_{41} = (\phi, 2, 1+3)$	+1	–1
$\vec{\pi}_{42} = (\phi, 3, 1+1+1)$	+1	–2
$\vec{\pi}_{43} = (\phi, 3+1, 1+1)$	+1	0
$\vec{\pi}_{44} = (5, \phi, 1)$	–1	–1
$\vec{\pi}_{45} = (5, 1, \phi)$	–1	1
$\vec{\pi}_{46} = (4, \phi, 2)$	–1	–1
$\vec{\pi}_{47} = (4, 2, \phi)$	–1	1
$\vec{\pi}_{48} = (\phi, 1+1+1, 3)$	+1	2
$\vec{\pi}_{49} = (\phi, 1+1, 3+1)$	+1	0
$\vec{\pi}_{50} = (\phi, 1, 3+1+1)$	+1	–2
$\vec{\pi}_{51} = (\phi, 3+1+1, 1)$	+1	2
$\vec{\pi}_{52} = (\phi, 2+2, 2)$	+1	1

$\vec{\pi}_{53} = (\phi, 2, 2+2)$	+1	–1
$\vec{\pi}_{54} = (\phi, 2, 1+1+1+1)$	+1	–3
$\vec{\pi}_{55} = (\phi, 1+1+1+1, 2)$	+1	3
$\vec{\pi}_{56} = (\phi, 2+1, 1+1+1)$	+1	–1
$\vec{\pi}_{57} = (\phi, 1+1+1, 2+1)$	+1	1
$\vec{\pi}_{58} = (\phi, 2+1+1, 1+1)$	+1	1
$\vec{\pi}_{59} = (\phi, 1+1, 2+1+1)$	+1	–1
$\vec{\pi}_{60} = (\phi, 1+1+1+1, 1+1)$	+1	2
$\vec{\pi}_{61} = (\phi, 1+1, 1+1+1+1)$	+1	–2
$\vec{\pi}_{62} = (\phi, 1+1+1, 1+1+1)$	+1	0
$\vec{\pi}_{63} = (\phi, 1, 1+1+1+1+1)$	+1	–4
$\vec{\pi}_{64} = (\phi, 1+1+1+1+1, 1)$	+1	4
$\vec{\pi}_{65} = (3, 2, 1)$	–1	0
$\vec{\pi}_{66} = (3, 1, 2)$	–1	0
$\vec{\pi}_{67} = (2, 3, 1)$	–1	0
$\vec{\pi}_{68} = (2, 1, 3)$	–1	0
$\vec{\pi}_{69} = (1, 2, 3)$	–1	0
$\vec{\pi}_{70} = (1, 3, 2)$	–1	0
$\vec{\pi}_{71} = (3, 1, 1+1)$	–1	–1
$\vec{\pi}_{72} = (3, 1+1, 1)$	–1	1
$\vec{\pi}_{73} = (2, 2+1, 1)$	–1	1
$\vec{\pi}_{74} = (2, 1, 1+2)$	–1	–1
$\vec{\pi}_{75} = (1, 1+1+1, 1+1)$	–1	1
$\vec{\pi}_{76} = (1, 1+1, 1+1+1)$	–1	–1
$\vec{\pi}_{77} = (1, 1, 1+1+1+1)$	–1	–3
$\vec{\pi}_{78} = (1, 1+1+1+1, 1)$	–1	3
$\vec{\pi}_{79} = (2, 1+1, 1+1)$	–1	0
$\vec{\pi}_{80} = (4, 1, 1)$	–1	0
$\vec{\pi}_{81} = (3, \phi, 3)$	–1	–1

$\vec{\pi}_{82} = (3,3,\phi)$	–1	1
$\vec{\pi}_{83} = (3,1+1+1,\phi)$	–1	3
$\vec{\pi}_{84} = (3,\phi,1+1+1)$	–1	–3
$\vec{\pi}_{85} = (2,2+2,\phi)$	–1	2
$\vec{\pi}_{86} = (2,\phi,2+2)$	–1	–2
$\vec{\pi}_{87} = (2,2+1+1,\phi)$	–1	3
$\vec{\pi}_{88} = (2,\phi,2+1+1)$	–1	–3
$\vec{\pi}_{89} = (2,\phi,2+2)$	–1	–2
$\vec{\pi}_{90} = (2,\phi,1+1+1+1)$	–1	–4
$\vec{\pi}_{91} = (1,1+1+1+1+1,\phi)$	–1	–4
$\vec{\pi}_{92} = (1,\phi,1+1+1+1+1)$	–1	–5
$\vec{\pi}_{93} = (1+2,3,\phi)$	+1	1
$\vec{\pi}_{94} = (1+2,\phi,3)$	+1	–1
$\vec{\pi}_{95} = (3+1,2,\phi)$	+1	1
$\vec{\pi}_{96} = (3+1,\phi,2)$	+1	–1
$\vec{\pi}_{97} = (3+1,1,1)$	+1	0
$\vec{\pi}_{98} = (4+1,1,\phi)$	+1	1
$\vec{\pi}_{99} = (4+1,\phi,1)$	+1	–1
$\vec{\pi}_{100} = (4,1+1,\phi)$	–1	2
$\vec{\pi}_{101} = (4,\phi,1+1)$	–1	–2
$\vec{\pi}_{102} = (3+1,1+1,\phi)$	+1	2
$\vec{\pi}_{103} = (3+1,\phi,1+1)$	+1	–2
$\vec{\pi}_{104} = (2+1,1+1+1,\phi)$	+1	3
$\vec{\pi}_{105} = (2+1,\phi,1+1+1)$	+1	–3
$\vec{\pi}_{106} = (2+1,1,2)$	+1	0
$\vec{\pi}_{107} = (2+1,2,1)$	+1	0
$\vec{\pi}_{108} = (1,2+1,2)$	–1	1
$\vec{\pi}_{109} = (1,2,2+1)$	–1	–1
$\vec{\pi}_{110} = (1,2+3,\phi)$	–1	2

$\vec{\pi}_{111} = (1, \phi, 2+3)$	–1	–2
$\vec{\pi}_{112} = (\phi, 4, 2)$	+1	1
$\vec{\pi}_{113} = (2+3, \phi, 1)$	+1	–1
$\vec{\pi}_{114} = (2, 1+3, \phi)$	–1	2
$\vec{\pi}_{115} = (2, \phi, 3+1)$	–1	–2
$\vec{\pi}_{116} = (1, 2+2+1, \phi)$	–1	3
$\vec{\pi}_{117} = (1, \phi, 2+2+1)$	–1	–3
$\vec{\pi}_{118} = (2+1, 1+1, 1)$	+1	1
$\vec{\pi}_{119} = (2+1, 1, 1+1)$	+1	–1
$\vec{\pi}_{120} = (1, 1+1, 2+1)$	–1	0
$\vec{\pi}_{121} = (1, 2+1, 1+1)$	–1	0

From this table we have;

$$N_V(0,11,6) = \omega(\vec{\pi}_{23}) + \omega(\vec{\pi}_{24}) + \omega(\vec{\pi}_{25}) + \omega(\vec{\pi}_{26}) + \omega(\vec{\pi}_{27}) + \omega(\vec{\pi}_{28}) + \omega(\vec{\pi}_{29}) + \omega(\vec{\pi}_{30}) + \omega(\vec{\pi}_{35}) + \omega(\vec{\pi}_{43}) + \omega(\vec{\pi}_{49}) + \omega(\vec{\pi}_{62}) + \omega(\vec{\pi}_{65}) + \omega(\vec{\pi}_{66}) + \omega(\vec{\pi}_{67}) + \omega(\vec{\pi}_{68}) + \omega(\vec{\pi}_{69}) + \omega(\vec{\pi}_{70}) + \omega(\vec{\pi}_{79}) + \omega(\vec{\pi}_{25}) + \omega(\vec{\pi}_{80}) + \omega(\vec{\pi}_{97}) + \omega(\vec{\pi}_{106}) + \omega(\vec{\pi}_{107}) + \omega(\vec{\pi}_{120}) + \omega(\vec{\pi}_{121})$$

= –1+1+1–1+1+1+1+1+1+1+1+1–1–1–1–1–1–1–1–1–1+1+1+1–1–1 = 1.

$$N_V(0,11,6) = 1 = \frac{P(6)}{11}$$, where n = 0 and k = 0.

Hence the result.

Conclusions

We verified that for any positive integral value of n in the relation $P(n) = \sum_{m=-\infty}^{\infty} N_V(m,n)$ and easily can find generating function for $N_V(m,n)$ in terms of various corresponding cranks of vector partitions.

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